Algebra (Basic Concepts & Formulas) for SSC & Railway Exams

Algebra (Basic Concepts & Formulas)

Important formulas in Algebra

1. $(a + b)^2 = a^2 + 2ab + b^2$; $a^2 + b^2 = (a + b)^2 - 2ab$
2. $(a - b)^2 = a^2 - 2ab + b^2$; $a^2 + b^2 = (a - b)^2 + 2ab$
3. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
4. $(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab - 2ac + 2bc$
5. $(a - b + c)^2 = a^2 + b^2 + c^2 - 2ab + 2ac - 2bc$
6. $a^2 - b^2 = (a + b)(a - b)$
7. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
8. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
9. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$; $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$
10. $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$; $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$
11. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$;

    If $a+b+c=0$, then $a^3 + b^3 + c^3 = 3abc$
12. $(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4)$
13. $(a - b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4)$
14. $a^4 - b^4 = (a - b)(a + b)(a^2 + b^2)$

15.$a^5 - b^5 = (a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$

16.If n is a natural number $a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + \ldots + b^{n-2}a + b^{n-1})$

17.$(a^m)(a^n) = a^{m+n}$

18.$(ab)^m = a^m b^m$

19.$(a^m)^n = a^{mn}$

20.$(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

21.$(a + b)^2 - (a - b)^2 = 4ab$

22.$(x + a)(x + b) = x^2 + (a + b)x + ab$

23.$(x+a)(x+b)(x+c) = x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc$

Imporant Tricks & concepts

1. If $x+1/x = a$, then $x^2+1/x^2 = a^2-2$
2. If $x-1/x = a$, then $x^2+1/x^2 = a^2+2$
3. If $x+1/x = a$, then $x^3+1/x^3 = a^3-3a$
4. If $x-1/x = a$, then $x^3-1/x^3 = a^3+3a$
5. If $x+1/x = a$, then $x^4+1/x^4 = a^4- 4a^2+2$
6. If $x+1/x = a$, then $x^4+1/x^4 = a^4- 4a^2+2$
7. If $x-1/x = a$, then $x^4+1/x^4 = a^4+ 4a^2+2$
8. If $x+1/x = a$, then $x^5+1/x^5 = a^5- 5a^3+5a$
9. If $x+1/x = a$, then $x^5- 1/x^5 = a^5+ 5a^3+5a$
10. If $x+1/x = a$, then $x^6+ 1/x^6 = a^6- 6a^4+9a^2-2$
11. If $x-1/x = a$, then $x^6+ 1/x^6 = a^6+ 6a^4+9a^2+2$

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Algebraic Expression

- An algebraic expression is a combination of constants and variables combined together with the help of the four fundamental signs

  Example: $x^3+4x^2- 8x +1$
- $x^3-4x^2- 8x -1$ is an algebraic expression containing three terms. The variable of this expression is x, coefficient of x^2 is 4, the coefficient of x is -8 and the constant is 1

Polynomials

- A polynomial is an arithmetic expression consisting of variables and constants that involves four fundamental arithmetic operations and non-negative integer exponents of variables.
- **Example**

S.No	Given expression	Polynomial / not a polynomial	Reason
1	$4x^2+2y^2+3y+6$	Polynomial	Non- negative integral power
2	$4x^{-4}+5x^4$	Not a polynomial	One of the powers is negative (–4)
3	$\sqrt{5}y^2$	Polynomial	Non- negative integral power

Degree of the Polynomial

- In a polynomial of one variable, the highest power of the variable is called the degree of the polynomial.
- In case of a polynomial of more than one variable, the sum of the powers of the variables in each term is considered and the highest sum so obtained is called the degree of the polynomial.

  Example: Find the degree of each term for the following polynomial and also find the degree of the polynomial $6ab^8+5a^2b^3c^2-7ab+4b^2c+2$

  Given polynomial is $6ab^8+5a^2b^3c^2-7ab+4b^2c+3$

  Degree of each of the terms

  $6ab^8$ has degree (1+8) =9

$5a^2b^3c^2$ has degree (2+3+2) =7

7ab has degree (1+1) =2

$4b^2c$ has degree (2+1) =3

The constant term 3 is always regarded as having degree Zero

The degree of the polynomial $6ab^8+5a^2b^3c^2-7ab+4b^2c+3$=the largest exponent in the polynomial=**9**

Types of Polynomials

Polynomial on the basis of number of terms

1	MONOMIAL	A polynomial having one term is called a monomial Examples: 6, 7m, 19ab
2	BINOMIAL	A polynomial having two terms is called a Binomial Examples : 5x+3,4a-2,10p+1
3	TRINOMIAL	A polynomial having three terms is called a Trinomial Example:$3x^2+4a+10$

Polynomial based on degree

1	CONSTANT	A polynomial of degree zero is called constant polynomial Examples :5,7,-14,547
2	LINEAR	A polynomial of degree one is called linear polynomial Examples :24x+2
3	QUADRATIC	A polynomial of degree two is called quadratic polynomial Example :$3x^2+5x+2$
4	CUBIC	A polynomial of degree three is called cubic polynomial Example : $x^3 -3x^2 + 4x - 2$

The general form of quadratic equation

x^2 - (sum of the roots) x + product of the roots = 0

Roots of Quadratic Equation

- For a quadratic equation $ax^2 + bx + c$ where $a \neq 0$, the roots will be given by the equation as $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Remainder Theorem

- If a polynomial p(x) of degree greater than or equal to one is divided by a linear polynomial (x–a) then the remainder is p(a), where a is any real number

  (i) If p(x) is divided by (x+a), then the remainder is p (– a)

  Example: Find the remainder when $p(x) = x^3 + 3x^2 + 3x + 1$ is divided by x+1.

  $p(x) = x^3 + 3x^2 + 3x + 1$

  f (–1) = (–1)3 +3(–1)2 +3(–1) +1 = –1+3–3+1 = 0

  Hence, the remainder is 0

Factor Theorem

- If p(x) is a polynomial of degree n > 1 and a is any real number, then

  (i) (x – a) is a factor of p(x), if p (a) = 0, and

  (ii) P (a) = 0, if (x – a) is a factor of p(x).

  Example: Show that (x + 2) is a factor of $x^3-4x^2-2x+20$

  $P(x)=x^3-4x^2-2x+20$

  $P(-2)=(-2)^3-4(-2)^2-2(-2)+20=0$

  P(-2)=0

  Therefore, (x + 2) is a factor of $x^3-4x^2-2x+20$