Algebra for SSC Exams

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1) If $5x+1/4x = 15$ then find the value of $16x^2+1/25x^2$

a) 245/3

b) 521/5

c) 712/5

d) 712/3

2) If $x – 1/x = 3$ then find the value of x^3+1/x^3

a) $11\sqrt{12}$

b) $10\sqrt{12}$

c) $11\sqrt{13}$

d) $10\sqrt{13}$

3) If $\sqrt{x} + 1/\sqrt{x} = 2$ then find the value of $5x^2 + 1/5x^2$

a) 27/5

b) 26/5

c) 24/5

d) 17/6

4) Solve: $4^{32} – 255(4^4+1) (4^8+1) (4^{16}+1)$

a) -1

b) 0

c) 1

d) None of the above

5) If $x + 1/x = 1$ then find the value of x^9+1/x^9

a) 2

b) 0

c) -2

d) None of the above

6) If $x = (\sqrt{15} + \sqrt{13})/ (\sqrt{15}-\sqrt{13})$ and $y = (\sqrt{15}-\sqrt{13})/ (\sqrt{15}+\sqrt{13})$ then find the value of $4x^2 – xy + 4y^2$

a) 658

b) 0

c) 125

d) 3127

7) If $a^3 – b^3 = 37$ and $a – b = 1$ then find the value of a^2+b^2

a) 23

b) 25

c) 24

d) 27

8) If $9x^2-6x+1=0$ then find the value of $81x^2 + (1/x)^2$

a) 27

b) 18

c) 36

d) -27

9) If $x + y + z = 9$ and $x^2 + y^2 + z^2 = 49$ then find the value of $\sqrt{(x^3 + y^3 + z^3 – 3xyz)}$

a) $18\sqrt{2}$

b) $3\sqrt{33}$

c) $9\sqrt{11}$

d) $19\sqrt{2}$

10) If $81a^4 + 225a^2b^2 + 625b^4 = 102$ and $9a^2 + 15ab + 25b^2 = 17$ then find the value of 3a+5b

a) $\sqrt{(55/3)}$

b) $\sqrt{(57/2)}$

c) $\sqrt{(55/2)}$

d) $\sqrt{(54/5)}$

11) If $(a + b – 5)^2 + (c – a – 6)^2 = 0$ then find the value of b + c

a) 11

b) 13

c) 12

d) None of the above

12) Simplify: $(a + b – c – d)^2 - (a – b + c – d)^2$

a) 4(ad + cd – ac)

b) 4(ab – cd + ac)

c) 4(ab + cd + ac+ bd)

d) 4(ab + cd – ac - bd)

13) If $1 – 216x^3 – 18x + (102 + a)x^2 = (1 – ax)^3$ then find the value of a

a) 6

b) 8

c) 7

d) None of the above

14) If $a^2-b^2+c^2=0$, then the value of $(a^6+c^6-b^6)/3(a^2+c^2)$

a) 1/3

b) a^2c^2

c) b^2c^2

d) 0

15) If a + b + c = 18, then find the value of

$(a-5)^3 + (b-6)^3 + (c-7)^3 - 3(a-5)(b-6)(c-7)$

a) 27

b) 247

c) 0

d) None of the above

16) If $x^4+x^{-4}=47$ then find the value of $(x+x^{-1})$

a) 3

b) 2

c) -3

d) 4

17) If $(16\sqrt{2}x^3+81\sqrt{3}y^3) \div (2\sqrt{2}+3\sqrt{3}) = (Ax+By+Cxy)$ then find the value of $5A-3B+C\sqrt{6}$

a) 67

b) 77

c) -77

d) 57

18) If $25x^2 + 49y^2 + 16z^2 +50=40x-42y+40z$, then the value of (14y + xz) will be

a) -4

b) 5

c) -5

d) 4

19) If (x + y) = 1 and (1/x) – 2y = 6/x then find the value of x^2+y^2

a) 6

b) 0

c) -5

d) 5

20) If $x=2\sqrt{5}+1/2\sqrt{5}$ and $y=2\sqrt{5}-1/2\sqrt{5}$ then find the value of

$\sqrt{(x^4 + y^4 - 2)}$

a) 4

b) -4

c) 0

d) None of the above

Algebra for SSC Exams

21) If $a^2 + b^2 + c^2 = 25$ and $ab + bc + ca = 12$ then find the value of $½(a + b + c)[(a – b)^2 + (b – c)^2 + (c – a)^2]$

a) -91

b) 87

c) 91

d) -87

22) Simplify: $(a + b)^3 + (a – b)^3 - 2a^3$

a) $6ab^2$

b) $8ab^2$

c) 0

c) (a^2+b^6)

23) If $x + y = 9$ and $xy = 10$ then find the value of $1/x^2+1/y^2$

a) 0.61

b) 0.061

c) 0.75

d) 0.94

24) If $a^3 + b^3 = 140$ and $a + b = 7$ then find the value of $(a-b)^2+ab$

a) 35

b) 25

c) 20

d) None of the above

25) If $x^2+1=3x$, then the value of $(x^4+x^{-2})/(x^2+3x+1)$ is:

a) -3

b) 0

c) 3

d) 1

26) If $x+1/x=5$ then find the value of x^3+1/x^3

a) 125

b) 139

c) 110

d) None of the above

27) If $x^2+y^2=23$ and $x – y = 6$ then find the value of xy

a) -6.5

b) -7.5

c) 8

d) 9

28) Simplify: $a/(a-b) +b/(b-a)$

a) ab

b) –ab

c) 1

d) None of the above

29) If a/b + b/a = - 1 then find the value of a^3-b^3

a) a + b

b) $a^2 + b^2$

c) 0

d) 1

30) If $16x^2+9/x^2=-24$ then find the value of $6x/(x^3+4x^2+x+3)$

a) 12

b) 24

c) 18

d) 16

31) If $3x – 3/x = 7$ then find the value of $x^3 – 1/x^3 – 7$

a) -432/27

b) 432/27

c) -343/27

d) 343/27

32) If $x = \sqrt[3]{37}$ and $y = \sqrt[3]{36}$ then find the value of $1/(x^2 + xy + y^2)$

a) x – y

b) x + y

c) $x^2 + y^2$

d) 1

33) If $a + 1/a = 1$ then find the value of $(a^6 + 1)/(a^6 – 1)$

a) 24

b) 13

c) 0

d) None of the above

34) If x-y=1 then find the value of x^3-y^3-3xy

a) 2xy

b) 1

c) -1

d) 0

35) If $x^4+1/x^4=167$ then, find the value of x-1/x

a) $2\sqrt{11}$

b) √15

c) √11

d) 5

36) If $3x^2+2x-6=0$ then find the value of $27x^6+8x^3+54x^5+36x^4-6$

a) 210

b) 216

c) 312

d) -216

37) If $x^2 – 3x + 1 = 0$ then find the value of $(x^2-1)/x$

a) 5

b) √5

c) √6

d) 6

38) If $x^3-1/x^3 = 0$, then find the value of $x^{81}- x^{15} -6x^{36} +7x^{42} +9x^{54}$

a) -9

b) 8

c) 10

d) 11

39) If $x=7+4\sqrt{3}$ and $xy=1$, then find the value of $((1/x^2)+(1/y^2))$

a) 194

b) 57

c) $85\sqrt{3}$

d) None of the above

40) If $x=11+2\sqrt{28}$ then find the value of $(x+1) / 4\sqrt{x}$

a) $(3+\sqrt{7})/(\sqrt{7}+\sqrt{4})$

b) $3\sqrt{2}$

c) $(\sqrt{3}+\sqrt{2})/(\sqrt{3}-\sqrt{2})$

d) $(\sqrt{3}-\sqrt{2})/(\sqrt{3}+\sqrt{2})$

41) Find the value of: $313^3-350^3+37^3$

a) 0

b) -12160065

c) 12160050

d) -12160050

42) If $3x^2+1=3x$, then find the value of $3x^4+x^2-2x+6$

a) -1

b) 1

c) 2

d) 5

43) If $x^2 -1/x=0$, then find the value of $x^{18}+3x^{13}-3x^{52}+13x^{99}$

a) -11

b) 14

c) 0

d) Can't be determined

44) If $8(x-1)^3-64(2x-3)^3+125(4x-5)^3 = -120(4x-5)(2x^2-5x+3)$,then find the value of x

a) 12/13

b) 15/14

c) 12/11

d) 0

45) If $x+1/x=4$, then find the value of $x^6+x^4-52x^3-14x^2$

a) -2

b) 1

c) -1

d) 0

46) If $3x-25= -3/x$ then find the value of $(x-1)/\sqrt{x}$

a) $\sqrt{(19/3)}$

b) $\sqrt{(23/3)}$

c) $\sqrt{(26/3)}$

d) None of the above

47) If $0.064x^3+0.125y^3=189$ and $0.16x^2+0.25y^2=9.20$ then find the value of $0.4x+0.5y$

a) 18

b) -9

c) 21

d) 0

48) If $a+b+c = 7$, $ab+bc+ca=12$, $a^3+b^3=19$, $b^3+c^3=37$ and $c^3+a^3=18$ then find the value of $3abc$

a) 20

b) 24

c) -18

d) -17

49) If a and b are the roots of the equation $x^2/3=(x-2)$ then what will be the equation whose roots are a^3/b^2 and b^3/a^2.

a) $4x^2-3x-24=0$

b) $4x^2+3x+24=0$

c) $4x^2+3x-24=0$

d) $4x^2-3x+24=0$

50) If $x^2 + y^2 – 4y – 6x = -13$ then find the value of x^4+y^5

a) 113

b) 124

c) 118

d) 120

ANSWERS

1) Answer: C

Solution:

$5x+1/4x=15$

Multiply by 4/5

$4x+1/5x=12$

Squaring on both sides

$(4x+1/5x)^2=144$

$16x^2+1/25x^2+2(4x)(1/5x)=144$

$16x^2+1/25x^2=144 – 8/5$

$16x^2+1/25x^2 = 712/5$

2) Answer: D

Solution:

$X – 1/x = 3$

Squaring on both sides

$(x – 1/x)^2=9$

$x^2 + 1/x^2 – 2 = 9$

$x^2 + 1/x^2 = 11$

Add 2 on both sides

$x^2 + 1/x^2 + 2 = 11 + 2$

$(x + 1/x)^2 = 13$

$X + 1/x = \sqrt{13}$

Cubing on both sides

$x^3+1/x^3+3(x+1/x)=13\sqrt{13}$

$x^3 + 1/x^3=13\sqrt{13}-3\sqrt{13}$

$x^3 + 1/x^3 = 10\sqrt{13}$

3) Answer: B

Solution:

$\sqrt{x}+ 1/\sqrt{x} = 2$

Squaring on both sides

$(\sqrt{x}+1/\sqrt{x})^2 = 4$

$x + (1/x) + 2 = 4$

$x + 1/x = 2$

By observing the above equation the value of x is 1

$5x^2+1/5x^2 = 5(1) +1/5(1) = 5+1/5 = 26/5$

4) Answer: C

Solution:

$= 4^{32} – 255(4^4+1) (4^8+1) (4^{16}+1)$

$= 4^{32} – (4^4-1)(4^4+1) (4^8+1) (4^{16}+1)$

Applying the formula $a^2-b^2 = (a+b)(a-b)$ for $(4^4-1)(4^4+1)$ terms

The above expression become

$= 4^{32} – (4^8-1) (4^8+1) (4^{16}+1)$

Similarly for $(4^8-1) (4^8+1)$

The above expression become

$= 4^{32} – (4^{16}-1) (4^{16}+1)$

$= 4^{32} – (4^{32}-1)$

$= 1$

5) Answer: C

Solution:

$X + 1/x = 1$

$x^2 – x + 1 = 0$ --------- (1)

As, $a^3 + b^3 = (a+b)(a^2 – ab + b^2)$

$a = x$ and $b = 1$

$x^3 + 1^3 = (x+1)(x^2-x+1)$ ---------- (2)

Substitute (1) in (2)

$x^3+1^3=(x+1)(0)$

$x^3 = -1$

Then => $x^9+1/x^9 = (x^3)^3+1/(x^3)^3$

$x^9+1/x^9= -1-1 = - 2$

Alternative method

$X + 1/x = 1$

Cubing on both sides

$X^3 + 1/x^3+3(X + 1/x)=1$

$X^3 + 1/x^3+3=1$

$X^3 + 1/x^3=-2$

Cubing on both sides

$X^9 + 1/x^9+3(X^3 + 1/x^3) =-8$

$X^9 + 1/x^9=-8+6$

$X^9 + 1/x^9=-2$

6) Answer: D

Solution:

$= 4x^2-xy+4y^2=4(x^2+y^2)-xy = 4[(x+y)^2-2xy]-xy = 4(x+y)^2-9xy$ –----- (1)

$x + y = [(\sqrt{15}+\sqrt{13})/ (\sqrt{15}-\sqrt{13})] + [(\sqrt{15}-\sqrt{13})/ (\sqrt{15}+\sqrt{13})]$

$= [(\sqrt{15}+\sqrt{13})^2+ (\sqrt{15}-\sqrt{13})^2]/((\sqrt{15})^2-(\sqrt{13})^2)$

$= [15+13+2(\sqrt{15})(\sqrt{13})+15+13-2(\sqrt{15})(\sqrt{13})]/2$

$=28$

$xy= [(\sqrt{15}+\sqrt{13})/(\sqrt{15}-\sqrt{13})][(\sqrt{15}-\sqrt{13})/ (\sqrt{15}+\sqrt{13})]=1$

$(1)=>4(28)^2-9 =3127$

7) Answer: B

Solution:

As, $(a-b)^3= a^3 - b^3 – 3ab(a – b)$ [Algebraic identity]

Put the values of a^3-b^3 and a-b in the above expression

$1^3 = 37 – 3ab(1)$

$ab = 12$

$a^3 – b^3= (a – b)(a^2+ab+b^2)$ [Algebraic identity]

put the values of a^3-b^3, a-b and ab in the above expression

$37=1(a^2+12+b^2)$

$a^2+b^2=25$

8) Answer: B

Solution:

$9x^2-6x+1=0$

÷ by x

$9x-6+1/x=0$

$9x+1/x=6$

Squaring on both sides

$(9x+1/x)^2=36$

$81x^2+1/x^2+18=36$

$81x^2+1/x^2=18$

9) Answer: B

Solution:

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$ [Algebraic identity]

Substitute the values of $(x + y + z)$ and $x^2 + y^2 + z^2$ in above expression

$9^2 = 49 + 2(xy + yz + zx)$

$(xy + yz + zx) = (81 – 49)/2 = 16$

$\sqrt{(x^3 + y^3 + z^3 – 3xyz)} = \sqrt{((x + y + z)(x^2 + y^2 + z^2 – xy – yz – zx))}$ [Algebraic identity]

Put the values of $(x + y + z)$, $(x^2 + y^2 + z^2)$, $(xy + yz + zx)$ in the above expression

$\sqrt{(x^3+y^3+z^3-3xyz)} = \sqrt{(9*(49-16)} = \sqrt{(9*33)}=3\sqrt{33}$

10) Answer: C

Solution:

$81a^4 + 225a^2b^2 + 625b^4 = 102$

The above expression is rewritten as $(3a)^4 + (9a^2)(25b^2) + (5b)^4 = 102$

$9a^2 + 15ab + 25b^2 = 17$

The above expression is rewritten as $(3a)^2 + 15ab + (5b)^2 = 17$ ------- (1)

$(a^4 + a^2b^2 + b^4) = (a^2 + ab + b^2)(a^2 – ab + b^2)$ [Algebraic identity]

$(3a)^4 + (9a^2)(25b^2) + (5b)^4 = [(3a)^2 + 15ab + (5b)^2][(3a)^2 – 15ab + (5b)^2]$

$102 = 17[(3a)^2 – 15ab + (5b)^2]$

$(3a)^2 – 15ab + (5b)^2 = 102/17 = 6$ ---------- (2)

Adding (1) and (2)

$(3a)^2+15ab+(5b)^2+(3a)^2-15ab+(5b)^2=17+6=33$

$2((3a)^2+(5b)^2)=33$

$(3a)^2+(5b)^2=33/2$ ---------- (3)

Subtracting (1) and (2)

$(3a)^2+15ab+(5b)^2-(3a)^2+15ab-(5b)^2=17-6=11$

$30ab=11$

$ab= 11/30$—---------- (4)

$(a)^2+(b)^2+2ab =(a+b)^2$ [Algebraic identity]

Therefore, $(3a+5b)^2=(3a)^2+(5b)^2+2(3a)(5b)$

$(3a+5b)^2= 33/2+11$

$(3a+5b)= \sqrt{(55/2)}$

11) Answer: A

Solution:

$(a+b-5)^2+(c-a-6)^2=0$

The above expression is of the form $x^2+y^2=0$

Since both terms are in square, the expression is valid only when both terms are 0

Therefore, $(a + b – 5) = 0$, $(c – a – 6) = 0$

a + b = 5 and c – a = 6

Adding both terms

a + b + c – a = 11

B + c = 11

12) Answer: D

Solution:

$= (a + b – c – d)^2 - (a – b + c – d)^2$

As, $(a + b + c + d)^2 = a^2 + b^2 + c^2 + d^2 + 2(ab + bc + ac + cd+ ad+ bd)$ [Algebraic identity]

$= (a^2+b^2+c^2+d^2+2ab-2bc+2cd-2ac-2ad-2bd) - (a^2+b^2+c^2+d^2-2ab-2bc-2cd+2ac-2ad+2bd)$

$= a^2+b^2+c^2+d^2+2ab-2bc+2cd-2ac-2ad-2bd - a^2-b^2-c^2-d^2+2ab+2bc+2cd-2ac+2ad-2bd$

= 4ab + 4cd – 4ac-4bd

=4(ab + cd – ac-bd)

13) Answer: A

Solution:

$1-216x^3-18x+ (102+a)x^2=(1-ax)^3$

Expand the RHS using the algebraic identity,

$(a-b)^3=a^3-b^3-3a^2b+3ab^2$

Then the given expression becomes,

$1-216x^3-18x+(102+a)x^2 = 1-(ax)^3-3(1)^2(ax)+3(1)(ax)^2$

Comparing the terms on both sides,

The coefficient of x^3, $216 = a^3$

a = 6

14) Answer: B

Solution:

$a^2-b^2+c^2=0$

$a^2+c^2= b^2$

Cubing on both sides

$(a^2+c^2)^3= b^6$

$a^6+c^6+3a^4c^2+3a^2c^4=b^6$

$a^6+c^6- b^6= 3a^4c^2+3a^2c^4$ ---------- (1)

As per Question

$= (a^6+c^6-b^6)/3(a^2+c^2)$

Put (1) in above expression

$= (3a^4c^2+3a^2c^4)/3(a^2+c^2)$

$= 3a^2c^2(a^2+c^2)/3(a^2+c^2)$

$= a^2c^2$

15) Answer: C

Solution:

The algebraic identity:

$a^3 + b^3 + c^3 – 3abc = (a + b + c)(a^2 + b^2 + c^2 – ab – bc – ca)$

So,$(a – 5)^3 + (b – 6)^3 + (c – 7)^3 – 3(a – 5)(b – 6)(c – 7) = (a – 5 + b – 6 + c – 7) ((a – 5)^2 + (b – 6)^2 + (c – 7)^2 - (a – 5)(b – 6) - (b – 6)(c – 7) - (c – 7)(a – 5))$

$=> (a – 5)^3 + (b – 6)^3 + (c – 7)^3 – 3(a – 5)(b – 6)(c – 7) = (a + b + c – 18)((a – 5)^2 + (b – 6)^2 + (c – 7)^2 - (a – 5)(b – 6) - (b – 6)(c – 7) - (c – 7)(a – 5))$

From question a + b + c = 18

$=> (a – 5)^3+(b-6)^3+(c-7)^3-3(a-5)(b-6)(c-7)=(18-18)((a-5)^2+(b-6)^2+(c-7)^2-(a-5)(b-6)- (b-6) (c -) - (c – 7)(a – 5))$

$=> (a-5)^3+ (b-6)^3+(c-7)^3-3(a-5)(b-6)(c-7) = 0$

16) Answer: A

Solution:

$x^4+x^{-4}=47$

$x^4+1/x^4=47$

Add 2 both sides

$x^4+1/x^4+2=49$

The above expression is of the form $a^2+b^2+2ab = (a + b)^2$ where a = x and b = 1/x

Then the above expression becomes

$(x^2+1/x^2)^2=7^2$

Taking square root on both sides

$(x^2+1/x^2) =7$

Again adding 2 both sides

$(x^2+1/x^2+2) =9$

$(x+1/x)^2=3^2$

Taking square root on both sides

$(x+1/x)=3$

17) Answer: C

Solution:

$(16\sqrt{2}x^3+81\sqrt{3}y^3) \div (2\sqrt{2}x+3\sqrt{3}y) = (Ax + By + Cxy)$

Expand the numerator by using the algebraic identity

$a^3+b^3= (a + b)(a^2 + b^2 – ab)$

$a = 2\sqrt{2}x$ and $b = 3\sqrt{3}y$

$[(2\sqrt{2}x + 3\sqrt{3}y)(8x^2 + 27y^2 – 6\sqrt{6}xy)]/(2\sqrt{2}x+3\sqrt{3}y) = (Ax + By + Cxy)$

$(8x^2+27y^2-6\sqrt{6}xy) = (Ax^2+By^2+Cxy)$

Comparing the coefficients of respective terms

A=8, B=27 and C=-6√6

5A-3B+C√6= 5(8)-3(27)-(6√6)√6

= 40 – 81 – 36 = - 77

18) Answer: C

Solution:

$25x^2+49y^2+16z^2+50=40x-42y+40z$

Rearrange the above expression

$\Rightarrow (5x)^2+(7y)^2+ (4z)^2-2(5x)(4)+2(7y)(3)-2(4z)(5)+16+9+25=0$

$\Rightarrow (5x)^2-2(5x)(4)+16+(7y)^2+2(7y)(3)+9+(4z)^2-2(4z)(5)+25=0$

$\Rightarrow [(5x)^2-2(5x)(4)+4^2]+[(7y)^2+2(7y)(3)+3^2]+[(4z)^2-2(4z)(5)+5^2]=0$

$\Rightarrow (5x-4)^2+ (7y+3)^2+ (4z-5)^2=0$

The above expression is 0 only when each term is 0

5x – 4 =0, 7y + 3 = 0, 4z – 5 = 0

X = 4/5, y = - 3/7 and z = 5/4

(14y + xz) = 14(-3/7) + (4/5)(5/4) = - 6 + 1 = - 5

19) Answer: A

Solution:

As, $x^2 + y^2 = (x + y)^2 – 2xy$ ---------- (1)

(1/x) – 2y = 6/x

1/x – 6/x = 2y

-5/x = 2y

2xy = - 5

Substitute the values of 2xy and x+y in (1)

$x^2 + y^2 = 1 - (-5) = 6$

20) Answer: A

Solution:

$= \sqrt{(x^4+y^4-2)}$

$= \sqrt{(x^2-y^2)^2}$

$= (x^2-y^2)$

$= ((2\sqrt{5} + (1/2\sqrt{5})^2 - (2\sqrt{5} - (1/2\sqrt{5})^2)$

The above expression is of the form,

$(a + b)^2 - (a – b)^2 = 4ab$

$A = 2\sqrt{5}$, $b = 1/2\sqrt{5}$

Then, $(2\sqrt{5} + 1/2\sqrt{5})^2 - (2\sqrt{5} – 1/2\sqrt{5})^2 = 4(2\sqrt{5})(1/2\sqrt{5}) = 4$

21) Answer: C

Solution:

As we know,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

Substitute the value of $a^2+b^2+c^2$ and ab + bc + ca in the above expression

$(a + b + c)^2 = 25 + 2(12) = 49$

$a + b + c = 7$

As we know,

$½(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2] = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$ [Algebraic identity]

Substitute the values of (a+b+c) and $(a^2+b^2+c^2-ab-bc-ca)$

$½(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2] = 7(25 – 12)$

$½(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2] = 91$

22) Answer: A

Solution:

$= (a + b)^3 + (a – b)^3 – 2a^3$

$= a^3+b^3+3a^2b+3ab^2+a^3-b^3-3a^2b+3ab^2-2a^3$

$= 6a^2b$

23) Answer: A

Solution:

$=1/x^2+1/y^2$

$= (x^2+y^2)/x^2y^2$

$= ((x + y)^2 – 2xy)/x^2y^2$

Substitute the values of (x + y) and xy

$= (9^2-20)/100 = 61/100 = 0.61$

24) Answer: C

Solution:

As, $a^3+b^3 = (a+b)(a^2-ab+b^2)$ ---------- (1)

As per question

$(a-b)^2+ab = a^2+b^2-2ab+ab = a^2+b^2-ab$

(1)=> $140=7(a^2+b^2-ab)$

$a^2+b^2-ab =20$

25) Answer: C

Solution:

Given:

$x^2+1=3x$

$x^2+1-3x=0$

÷ by x

$x + 1/x = 3$

Cubing on both sides

$(x+1/x)^3 = x^3+1/x^3+3(x)(1/x)(x+1/x)$

Put the value of (x+1/x) in above expression

$27= x^3+1/x^3+9$

$x^3+1/x^3=18$

According to the question,

$= (x^4+x^{-2})/(x^2+3x+1)$

$= (x^4+x^{-2})/(x^2+3x+1-3x+3x)$ {add and subtract 3x}

$=(x^4+x^{-2})/(x^2+1-3x+6x)$

Put the value of x^2+1-3x in above expression

$= (x^4+1/x^2)/(6x)$

$=x(x^3+1/x^3)/6x$

$= (x^3+1/x^3)/6$ --- (1)

Put the values of (x^3+1/x^3) in (1)

$(x^4+x^{-2})/(x^2+3x+1) =18/6=3$

26) Answer: C

Solution:

x+1/x=5

Cubing on both sides

$(x+1/x)^3=125$

As, $(a+b)^3 = a^3+b^3+3ab(a+b)$

$x^3+1/x^3+3(x+1/x)=125$

$x^3+1/x^3=125-15=110$

27) Answer: A

Solution:

$(x-y)^2=x^2+y^2-2xy$

Substitute the value of x^2+y^2 and x-y in above expression

$6^2=23-2xy$

36-23=-2xy

-13/2 =xy

xy=-6.5

28) Answer: C

Solution:

= a/(a-b) + b/(b-a)

= a/(a-b) – b/(a-b)

= (a – b)/(a – b) =1

29) Answer: C

Solution:

a/b + b/a = - 1

$(a^2+b^2)/ab = -1$

$a^2+b^2 = -ab$

$a^2 + b^2 + ab = 0$

As, $a^3-b^3= (a-b)(a^2+b^2+ab)$ --- Algebraic identity

Substitute the values in the above expression then

$a^3-b^3= (a-b)(0)=0$

30) Answer: B

Solution:

Given:

$16x^2 + 9/x^2 = -24$

$16x^2 + 9/x^2 + 24 = 0$

$(4x)^2 + (3/x)^2 + 2(4x)(3/x) = 0$

The above expression is of the form $a^2 + b^2 + 2ab = (a+b)^2$

So, $(4x)^2+(3/x)^2+2(4x)(3/x)=0$

$(4x+3/x)^2 = 0$

$4x + 3/x = 0$

$x^2 = -3/4$

According to the question

$= 6x/(x^3+4x^2+x+3)$

$= 6x/x(x^2+4x+1+3/x)$

$=6/(x^2+1+4x+3/x)$

Put the value of $4x+3/x$ in the above expression

$=6/(x^2+1+0)$

$=6/(-3/4+1)$ {Since $x^2=-3/4$}

$= 6/(1/4) =24$

31) Answer: D

Solution:

$3x – 3/x = 7$

$\div$ by 3 => $x – 1/x = 7/3$

Cubing on both sides

$(x – 1/x)^3 = 343/27$

$x^3-1/x^3-3(x-1/x)=343/27$

Put the value of $x-1/x$ in the above expression

$x^3-1/x^3-3(7/3)=343/27$

$x^3-1/x^3-7=(343/27)$

32) Answer: A

Solution:

$=1/(x^2 + xy + y^2)$

Multiply and divide by (x – y)

= (x – y)/[(x – y)(x^2 + xy + y^2)]

= (x – y)/(x^3 – y^3)

= (x – y)/(37 – 36) = x – y

33) Answer: C

Solution:

Given:

a + 1/a = 1

Squaring on both sides

a^2+1/a^2+2 = 1

a^2+1/a^2+1=0 ------- (1)

According to the question,

= (a^6-1)/(a^6+1)

= a^3(a^3-1/a^3)/a^3(a^3+1/a^3)

= (a^3-1/a^3)/(a^3+1/a^3)

Expand the above expression using the algebraic identity

= [(a-1/a)(a^2+1+1/a^2)]/[(a+1/a)(a^2-1+1/a^2)] ---(2)

Put (1) in (2)

(2)=> [(a-1/a)(0)]/[(a+1/a)(a^2-1+1/a^2)]=0

34) Answer: B

Solution:

X-y=1---→1

Take cube root on both sides

$(X-Y)^3=1^3$

X^3-Y^3-3xy(x-y) =1 (x-y=1 from equation 1)

X^3-y^3-3xy=1

35) Answer: C

Solution:

x^4+1/x^4=167

Add 2 on both sides

x^4+1/x^4+2=167+2

(x^2+1/x^2)^2=169

(x^2+1/x^2)=13

Subtract 2 on both sides

(x^2+1/x^2-2)=13-2

$(x-1/x)^2$=11

x-1/x=√11

36) Answer: A

Solution:

$3x^2+2x-6=0$

$3x^2+2x=6$

Cubing on both sides

$(3x^2+2x)^3=216$

$27x^6+8x^3+54x^5+36x^4=216$

Subtract 6 on both sides

$27x^6+8x^3+54x^5+36x^4-6=210$

37) Answer: B

Solution:

$= (x^2-1)/x$

The above expression can be rewritten as

$= x-1/x$ ----------→ (1)

Given:

$x^2-3x+1=0$

$x-3+1/x=0$

$x+1/x=3$

As, algebraic identity $(x-1/x)^2=(x+1/x)^2-4$

Substitute the value of $x+1/x$ in above expression

$(x-1/x)^2=(3)^2-4 = 5$

$(x – 1/x) = \sqrt{5}$

38) Answer: C

Solution:

Given:

$x^3-1/x^3=0$

$x^6=1$

According to the question

$= x^{81}- x^{15} -6x^{36} +7x^{42} +9x^{54}$

$= x^3x^{78}-x^{18}/x^3-6x^{36} +7x^{42} +9x^{54}$

$= x^3(x^6)^{13}-(x^6)^3/x^3-6(x^6)^6+7(x^6)^7+9(x^6)^9$

Put the value of x^6 in above expression

$= x^3(1)^{13}-(1)^3/x^3-6(1)^6+7(1)^7+9(1)^9$

$= x^3-1/x^3-6+7+9$

$= 0+10 =10$ (As, $x^3-1/x^3=0$)

39) Answer: A

Solution:

Given:

$xy=1$

$y=1/(7+4\sqrt{3})$

Taking complex conjugate

$y= (7-4\sqrt{3})/((7+4\sqrt{3})(7-4\sqrt{3}))$

$=7-4\sqrt{3}/(49-48)=(7-4\sqrt{3})$

$y= (7-4\sqrt{3})$

Question:

$(1/x^2 + 1/y^2) = (x^2+y^2)/(xy)^2$

$= [(x+y)^2-2xy]/(xy)^2$ ---(1)

$xy=(7+4\sqrt{3})\ (7-4\sqrt{3})= 49-48=1$

$x+y=(7+4\sqrt{3})+(7-4\sqrt{3})=14$

$(1)=> (14^2-2)/1 = 194$

40) Answer: A

Solution:

$x =11+2\sqrt{28}$

$= 7+4+2\sqrt{28}$

$= (\sqrt{7})^2+(\sqrt{4})^2+2*\sqrt{7}*\sqrt{4} = (\sqrt{7}+\sqrt{4})^2$

$\sqrt{x} = \sqrt{7} +\sqrt{4}$

$x+1 = 12+2\sqrt{28}$

$(x+1)/4\sqrt{x} = (12+2\sqrt{28})/4(\sqrt{7}+\sqrt{4})$

$= (3+\sqrt{7})/(\sqrt{7}+\sqrt{4})$

41) Answer: D

Solution:

$= 313^3-350^3+37^3$

As algebraic identity,

$a^3+b^3+c^3-3abc= (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

Since, a+b+c = 313-350+37=0

Then, $a^3+b^3+c^3-3abc=0$

$a^3+b^3+c^3=3abc$

$313^3+(-350)^3+37^3=3(313)(-350)(37)=-12160050$

42) Answer: D

Solution:

$3x^2+1=3x$

$3x^2+1-3x=0$ ------ (1)

Multiply by x

$1 => 3x^3+x-3x^2=0$ ---- (2)

Multiply (2) by x

$2 => 3x^4+x^2-3x^3=0$ ----(3)

Adding 1, 2&3

$3x^2+1-3x+3x^3+x-3x^2+3x^4+x^2-3x^3=0$

$1-2x+x^2+3x^4=0$

$3x^4+x^2-2x+1=0$

Add 5 on both sides

$3x^4+x^2-2x+6=5$

43) Answer: B

Solution:

$x^2-1/x=0$

$x^3-1=0$

$x^3=1$

$=x^{18}+3x^{13}-3x^{52}+13x^{99}$

$=(x^3)^6+3x(x^3)^4-3x(x^3)^{17}+13(x^3)^{33}$

$= (1)^6+3x(1)^4-3x(1)^{17}+13(1)^{33}$

$=1+3x-3x+13$

$= 14$

44) Answer: B

Solution:

$8(x-1)^3-64(2x-3)^3+125(4x-5)^3 = -120(4x-5)(2x^2-5x+3)$

$[2(x-1)]^3+[-4(2x-3)]^3+[5(4x-5)]^3 = 3*2(x-1)*(-4(2x-3))*5(4x-5)$

The above expression is of the form

$a^3+b^3+c^3 = 3abc$

If $a^3+b^3+c^3 = 3abc$ then $a+b+c=0$

$a=2(x-1)$, $b=-4(2x-3)$, $c=5(4x-5)$

$a+b+c=2x-2-8x+12+20x-25=0$

$14x-15=0$

$x=15/14$

45) Answer: A

Solution:

If $x+1/x=4$ --- (1)

Squaring on both sides, $(x+1/x)^2=16$

$x^2+1/x^2+2=16$

$x^2+1/x^2=14$

Cubing (1) => $(x+1/x)^3=64$

$x^3+1/x^3+3(x)(1/x)(x+1/x)=64$

$x^3+1/x^3=52$

According to the Question

$=x^6+x^4-52x^3-14x^2$

Substitute the value of 52 and 14 in above expression

$=x^6+x^4-(x^3+1/x^3)x^3-(x^2+1/x^2)(x^2)$

$=x^6+x^4-x^6-1-x^4-1=-2$

46) Answer: A

Solution:

3x-25= -3/x

3x+3/x=25

Divide by 3

x+1/x=25/3

Subtract 2 both sides

x+1/x-2=(25/3)-2

$(\sqrt{x}-1/\sqrt{x})^2 = 19/3$

$\sqrt{x}-(1/\sqrt{x}) = \sqrt{(19/3)}$

$(x-1)/\sqrt{x} = \sqrt{(19/3)}$

47) Answer: C

Solution:

Given:

$0.064x^3+0.125y^3=189$

$(0.4x)^3+(0.5y)^3=189$ ---(1)

$0.16x^2+0.25y^2=9.20$

$0.16x^2+0.25y^2=9+0.20$

$(0.4x)^2+(0.5y)^2-0.20=9$ ---(2)

As, $a^3+b^3= (a+b)(a^2-ab+b^2)$ ---(3)

By analysing the (1),(2) and(3)

a=0.4x and b=0.5y

(3)=> $(0.4x)^3+(0.5y)^3=(0.4x+0.5y)[(0.4x)^2+(0.5y)^2-0.20]$

Put the respective values in the above expression

189= (0.4x+0.5y)(9)

(0.4x+0.5y)=21

48) Answer: C

Solution:

$(a + b + c)^2=a^2+b^2+c^2+2(ab+bc+ca)$ ---------- (1)

$49 = a^2+b^2+c^2+2(12)$

$a^2+b^2+c^2 = 25$

$a^3+b^3+b^3+c^3+c^3+a^3=19+37+18=74$

$2(a^3+b^3+c^3) =74$

$a^3+b^3+c^3 =37$

As, $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$---------- (2)

Substitute the respective values

37-3abc=7*(25-12)

3abc = 37 – 91

3abc = - 54

Abc = -18

49) Answer: B

Solution:

$x^2/3=(x-2)$

$x^2=3x-6$

$x^2-3x+6=0$

Sum of the roots (a+b) = 3

Product of the roots (ab) = 6

According to the Question,

The roots of the required equation are a^3/b^2 and b^3/a^2

Product of roots = $a^3/b^2 * b^3/a^2 = ab=6$

Sum of roots = $a^3/b^2 + b^3/a^2$

$= (a^5+b^5)/a^2b^2$ ------ (1)

$(a^2+b^2)(a^3+b^3)=a^5+b^5+b^2a^3+a^2b^3$

$(a^2+b^2)(a^3+b^3)=a^5+b^5+b^2a^2(a+b)$------(2)

$a^2+b^2 = (a+b)^2-2ab=9-12=-3$

$a^3+b^3 = (a+b)^3-3ab(a+b)=27-54=-27$

(2) $=>(-3)(-27)=a^5+b^5+(6)^2*3$

$a^5+b^5=81-108=-27$

(1)=>Sum of roots = -27/36 =-3/4

Required equation:

x^2-(sum of the roots)x+product of roots = 0

$x^2-(-3/4)x+6=0$

$4x^2 + 3x + 24 = 0$

50) Answer: A

Solution:

$x^2+y^2-4y-6x=-13$

$x^2+y^2-4y-6x+13=0$

$x^2-6x+9+y^2-4y+4=0$

$(x-3)^2+(y-2)^2= 0$

The above equation will be satisfied only when both terms are **0**.Since both terms are in square.

Therefore, x-3=0 and y-2=0

x=3 and y=2

According to the Question:

$x^4+y^5=3^4+2^5=81+32=113$

Algebra for SSC Exams